ENGINEERING MATHEMATICS

(For ESE & GATE Exam)

(CE, ME, PI, CH, EC, EE, IN, CS, IT)

Salient Features :

- 280 topics under 32 chapters in 8 units
- 640 Solved Examples for comprehensive understanding
- 1566 questions from last 29 years of GATE & ESE exams with detailed solutions
- Only book having complete theory on ESE & GATE Pattern
- Comprising conceptual questions marked with '*' for quick revision

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PREFACE

Science and technology has contributed greatly to the progress of the human race, and thus has made our life quite easier and comfortable. Engineers have played a vital role in the advancements of the technological world. However, with advancements in technology, there come challenges. It is here that the role of Engineering Mathematics comes in play to address the technological challenges posed by the modern era.

Engineering Mathematics is the art of applying mathematics to complicated real-world problems. It is a mix of mathematical theory, practical engineering and scientific computing.

Keeping in mind the importance of Engineering Mathematics skill in modern engineers, in the year 2017, the Union Public Service Commission (UPSC) introduced Engineering Mathematics as a common paper in the syllabus for Engineering Services Examination (ESE), and technical paper for Electrical (EE) stream. It has already been given a weightage of 15% in the Graduate Aptitude Test in Engineering (GATE)

With an objective to develop an ESE or GATE aspirant's numerical abilities and calculation skills, IES Master has come up with this Engineering Mathematics book that brings them face to face with 280 topics under 32 chapters in 8 units, along with previous years questions from GATE (last 29 years), and ESE (last 5 years) and their detailed solutions. Equipped with all this, students can easily decide, how much time to allocate on each chapter based on the number of questions asked in that particular exam.

It is the only book in the market that includes complete theory exactly on the basis of ESE & GATE exams pattern. After each topic, the book includes more than 640 solved examples for concept building & easy learning. To save students time during revision, all the previously asked conceptual questions are marked with '*' symbol.

Our special thanks to the entire IES Master team for their continuous support in bringing out the book. We strongly believe that this book will help students in their journey to success. Suggestions from students, teachers & educators for further improvement in the book are always welcome.

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UNIT 1 : LINEAR ALGEBRA

UNIT 4 : COMPLEX ANALYSIS

UNIT 5 : DIFFERENTIAL EQUATIONS

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UNIT 7 : TRANSFORM THEORY

UNIT 8 : PROBABILITY & STATISTICS

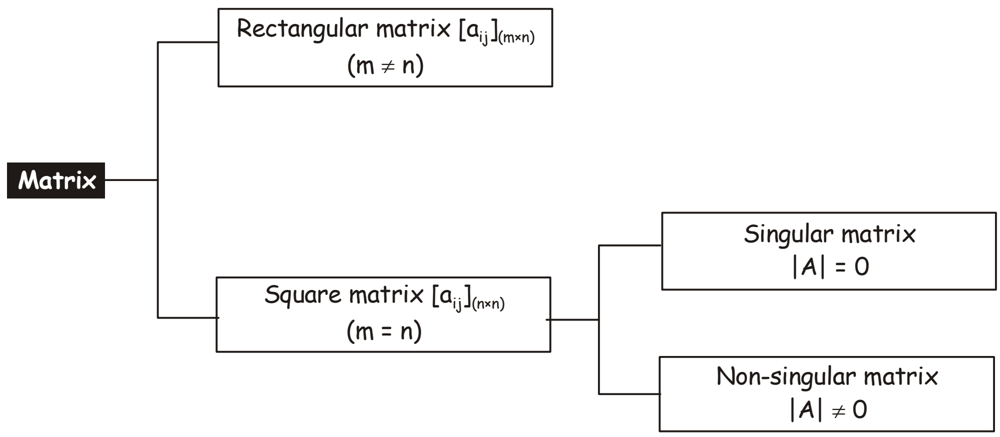
Algebra of Matrices 1.1

DEFINITION OF MATRIX

Matrix is a convenient way of storing information in form of m horizontal rows and n vertical columns.

Matrix can be represented either $A = (a_{ij})_{m\times n}$ or $A = [a_{ij}]_{m\times n}$

TYPES OF MATRICES



Rectangular Matrices:

1. **Row Matrix:** A matrix $A = [a_{ij}]_{m\times n}$ such that $m = 1$ and $n > 1$, matrix is known as row matrix.

   e.g. $A = [a_{ij}]_{1\times n} = [a_{11}, a_{12}, ... a_{1n}]$ or $[a_1, a_2, ..., a_n]$ is a row matrix of order n or matrix of order $1 \times n$.

2. **Column Matrix:** A matrix $A = [a_{ij}]_{m\times n}$ such that $m > 1$ and $n = 1$, matrix is known as row matrix.

   e.g. $A = [a_{ij}]_{n\times 1} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ or $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$ is a column matrix of order m or matrix of order $m \times 1$.

3. **Null Matrix or Zero Matrix:** A matrix $A = [a_{ij}]_{m\times n}$ such that $[a_{ij}] = 0$ $, \forall\, i, j \in N$ is called a zero matrix or Null Matrix i.e.

$$O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4. **Horizontal Matrix :** A matrix $A = [a_{ij}]_{m\times n}$ such that $m < n$, matrix is called as horizontal matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}_{3\times4}$$

5. **Vertical Matrix :** A matrix $A = [a_{ij}]_{m\times n}$ such that m > n, matrix is called as vertical matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}_{4\times3}$$

Square Matrix:

A matrix in which the number of rows is equal to the number of columns is called a square matrix i.e. $A = (a_{ij})_{m\times n}$ is a square matrix if and only if m = n. A matrix

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$ is a square matrix of order 3. The elements a_{11}, a_{22}, a_{33} of the above square matrix are called its diagonal elements and the diagonal containing these elements is called the principal diagonal or leading diagonal or main diagonal.

Trace of Matrix: The sum of the diagonal elements of a square matrix is called **trace** of the matrix.

1. **Diagonal Matrix:** A square matrix is called diagonal matrix if all its non-diagonal elements are zero i.e. in general a matrix $A = (a_{ij})_{n\times n}$ is called a diagonal matrix if $a_{ij} = 0$ for $i \neq j$;

   For example $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a diagonal matrix of order 3´.

2. **Upper Triangular Matrix:** A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ whenever i > j. Thus in an upper triangular matrix all the elements below the principal diagonal are zero. For example,

$$A = \begin{bmatrix} 1 & 2 & 4 & 2 \\ 0 & 3 & -1 & 5 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 4 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

   are 4× 4 and 3×3 upper triangular matrices respectively.

3. **Lower Triangular Matrix :** A square matrix of $A = [a_{ij}]$ is called a lower triangular matrix if $a_{ij} = 0$ whenever i < j. Thus in a lower triangular matrix all the elements above the principal diagonal are zero. For example, $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 4 & 2 & 3 & 0 \\ 5 & 4 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$ are 4×4 and 2×2 lower triangular matrices respectively.

4. **Scalar Matrix:** If all the elements of a diagonal matrix of order n are equal, i.e. if $a_{ii} = k \,\forall\, i$, then the matrix is called a scalar matrix, i.e.

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$ is a scalar matrix of order 3.

5. **Unit or Identity Matrix:** A square matrix is called a unit matrix or identity matrix if all the diagonal elements are unity and non-diagonal elements are zero. e.g.

$$I_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, I_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

are identity matrices of order 3×3 and 2×2 respectively.

6. **Sub matrix :** A matrix obtained from a given matrix, say A = $(a_{ij})_{m\times n}$ by deleting some rows or columns or both is called a sub matrix of A. For example if $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 5 & 1 \\ 7 & 8 & 0 & 2 \\ 1 & 7 & 2 & 3 \end{bmatrix}$ then the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 7 & 8 & 0 \end{bmatrix}$ and $\begin{bmatrix} 3 & 5 \\ 8 & 0 \\ 7 & 2 \end{bmatrix}$ are sub matrices of A.

7. **Equal matrices :**

   Two matrices are said to be equal if :

   (i) They are of the same order.

   (ii) The elements in the corresponding positions are equal.

   Thus if $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$. Then A = B

   In general if $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{m\times n}$ are matrices each of order m×n and $a_{ij} = b_{ij}$ for all i and j then A = B.

PRODUCT OF MATRIX BY A SCALAR (OR CONSTANT)

Let $A = [a_{ij}]_{m\times n}$ be a matrix of order m×n and k is a constant, then their product is matrix $kA = [ka_{ij}]_{m\times n}$ i.e. every element of A is multiplied by k. For example, if $A = \begin{bmatrix} 2 & -1 & 0 \\ 4 & 5 & -3 \end{bmatrix}$. Then we have $4A = \begin{bmatrix} 8 & -4 & 0 \\ 16 & 20 & -12 \end{bmatrix}$

ADDITION AND SUBTRACTION OF MATRICES

Let A and B be two matrices of the same order, then their sum A + B is defined as the matrix each element of which is the sum of the corresponding elements of A and B.

In general if $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{m\times n}$ then their sum is defined by the matrix.

$$C = A + B = (C_{ij})_{m\times n}$$

where $c_{ij} = a_{ij} + b_{ij}$ and i = 1, 2, ...m and j = 1, 2, 3,.......,n

If $A = \begin{bmatrix} 4 & 2 & 5 \\ 11 & 13 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$

Then
$$A + B = \begin{bmatrix} 4+1 & 2+0 & 5+2 \\ 11+3 & 13+1 & -6+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 7 \\ 14 & 14 & -2 \end{bmatrix}$$

Similarly, if A and B are two matrices of the same order, then their difference is defined by

$$A - B = \begin{bmatrix} 4-1 & 2-0 & 5-2 \\ 11-3 & 13-1 & -6-4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ 8 & 12 & -10 \end{bmatrix}$$

Properties of Matrix Addition

(i) Matrix addition is commutative :