ENGINEERING APTITUDE

(QUANTITATIVE APTITUDE AND ANALYTICAL ABILITY)

(For ESE GATE & PSUs Exam)

(CE, ME, PI, CH, EC, EE, IN, CS, IT)

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PREFACE

Union Public Services Commission (UPSC) in its quest for best engineering minds looks for the very basic pride of an engineer, which it tests through quantitative and analytical abilities of candidates. The profession itself calls for putting candidates in situations, both human and technical, where things are tied into hundred knots. As an engineer one is expected to think critically, detect systematic themes while analysing data, and achieve thoroughness with accuracy in deriving solutions under challenging circumstances.

To test these qualities in an ESE aspirant, in the year 2016, UPSC introduced Engineering Aptitude as a part of the syllabus for the common paper of ESE in 2016. With an objective to develop these abilities, IES Master has come up with the revised and updated **Fifth edition** of the book Engineering Aptitude that acquaints an ESE aspirant to thousands of problems under various sub-heads such as probability, polynomials, speed-time, work-time, clock and calendar, as well as geometry and measurements that they might come across during their professional career. Covering more than **170 topics under 25 chapters in 5 units**, this book is an effort by IES Master to offer the complete theory of ESE syllabus along with previous years questions from **UPSC (last 39 years), GATE (last 12 years), and ESE (last 5 years) to the ESE aspirants**.

As you flip through the pages of this book, it captures your imagination with subtleness, and exposes you to more than **1,650$^+$ problems**, enough to give your pen the required strength to take on any competitive exams including ESE, GATE and PSUs.

Having gone through the clarity and conciseness offered in this revised edition, we hope that as your fingers follow your command, the brain will engineer solutions no matter how difficult the challenge is.

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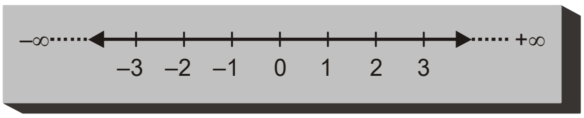
1.1 NUMBER SYSTEM

NUMBER LINE

Number line is a line on which all the positive and negative numbers can be represent in a sequence. It stretches from negative infinity to positive infinity.



DEFINITION OF VARIOUS TYPES OF NUMBERS

Natural Numbers

Counting Numbers 1, 2, 3, 4, ... are called Natural Numbers. The symbolic representation is N, i.e.,

N = {1, 2, 3, 4, 5, ...}.

Whole Numbers

All the natural numbers together with '0' are called Whole Numbers and the symbolic representation is W, i.e., W = {0, 1, 2, 3, 4, ...}

Integers

An integer is a number that can be written without a fractional component, it is represented by 'Z'.
Integers are further classified into positive integers (2, 4, 5 etc.), zero (0) and negative integers (–2, –5 etc.).

Rational Numbers

A number which can be expressed in the form of $\frac{p}{q}$ where, p and q are integers and $q \neq 0$, is called a rational number.

For example : *Any integer number is a rational number since, it can be written as the ratio of two integer numbers, one the integer number itself and another number is 1.*

Other examples of rational number are $\frac{2}{3}, -\frac{3}{7}, \ldots$ etc.

> ***Note :*** *A decimal represents a rational number if and only if it has a finite number of digits. But recurring decimals are exceptions as they are also assumed as Rational Numbers, i.e., all recurring decimals are rational numbers.*

Irrational Numbers

A real number, which is not rational, is called **irrational number**. An irrational number has non-terminating and non-recurring decimal part.

Between any two numbers, there are infinite numbers of irrational numbers.

Examples of irrational numbers are : $\sqrt{3}, \sqrt{5}, \sqrt[3]{7}, \sqrt[4]{11}$

Numbers π and e are also irrational number because both have non-terminating and non-recurring decimal part.

π = 3.14159265358979… and e = 2.71828182859045… where, e is called Euler's number.

> ***Note :** Any terminating **or** recurring decimal is a rational number. Any non-terminating **and** non-recurring decimal is an irrational number.*

Example 1

Which one of the following is not a rational number?

(a) $\frac{3}{8}$ (b) $-\frac{111}{23}$ (c) $\sqrt{2}$ (d) None of these

Sol. (c)

The number is option (a) and (b) are rational numbers, as they are the ratio of two integers. The number $\sqrt{2}$ is non-recurring, so it is not a rational number.

Real Numbers

The real numbers include all the measuring numbers. The symbol for the real number is R. **All the numbers which can be represented on the number line are called real numbers.**

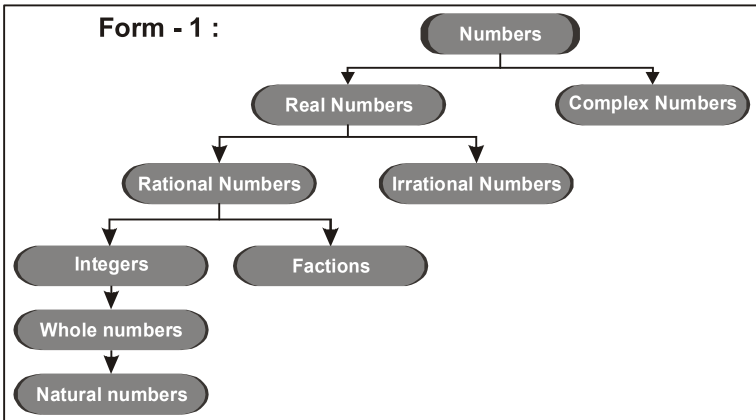
Complex Numbers

All the numbers that can be represented in a + ib form where a & b are real numbers and i = $\sqrt{-1}$ are called Complex Numbers.

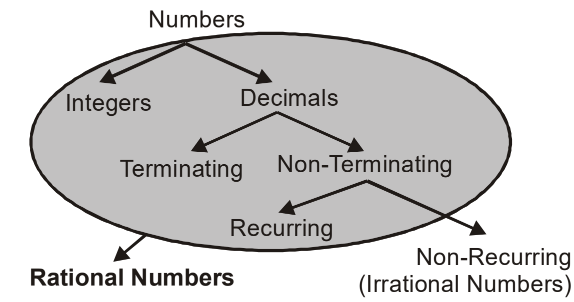
$$C = \{a + ib; a, b \in R \ \& \ i^2 = -1\}$$

VARIOUS FORMS OF NUMBER TREE

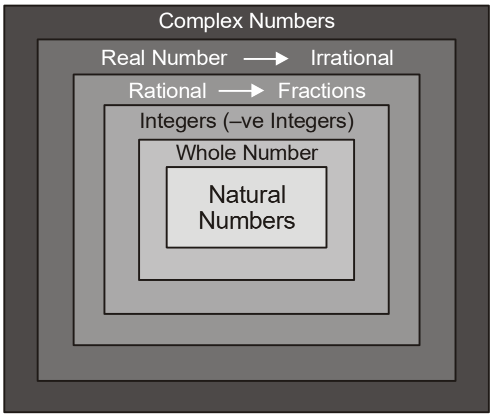
Form - 1 :



Form - 2 :



Form - 3 :



Example 2

Consider the following statements :

I. Every natural number is a real number.

II. Every real number is a rational number.

III. Every integer is a real number.

IV. Every rational number is a real number.

Which of the above statements are correct?

(a) I, II and III (b) I, III and IV (c) II and III (d) III and IV

Sol. (b)

From the number tree, given above, all natural numbers are real numbers but its converse is not true. So, statement (I) is true.

Every real number is not a rational number, some may be irrational numbers. Hence, statement (II) is wrong.

Similarly, from number tree, we can say about statement (III) and (IV) that both the statements are true.

Recurring Decimals

A decimal in which a digit or a set of digits is repeated continuously is called a recurring decimal. For representing recurring decimal, we place bar on the repeated numbers.

For example : (i) The number $0.247632476324763\ldots$ can be represented as $0.\overline{24763}$

(ii) and similarly, the number $0.1555\ldots$ can be represented as $0.1\overline{5}$

Example 3

Express the recurring decimal $0.\overline{230}$ in the form of a fraction.

Sol.

The given decimal can be written as $0.\overline{230} = 0.230230230\ldots$ …(i)

As the bar is placed on **three digits**, so, we will multiply the above equation by 10^3

$$0.\overline{230} \times 10^3 = 230.230230\ldots \quad \ldots(ii)$$

From equation (i) and (ii), we can write

$$(10^3 - 1)\,0.\overline{230} = 230 \quad \text{or} \quad 0.\overline{230} = \frac{230}{10^3 - 1} = \frac{230}{999}$$

Example 4

The value of $1.\overline{34} + 4.1\overline{2}$ is

(a) $\frac{133}{90}$ (b) $\frac{371}{90}$ (c) $5\frac{219}{990}$ (d) $5\frac{461}{990}$

Sol. (d)

The decimal $1.\overline{34}$ can be written as $1.\overline{34} = 1.343434\ldots$ …(i)

As the bar is placed on two digits after decimal point, so, we will multiply the above equation by 10^2

$$1.\overline{34} \times 10^2 = 134.3434\ldots \quad \ldots(ii)$$

From equation (i) and (ii)

$$(10^2 - 1)\,1.\overline{34} = 133 \qquad \therefore\ 1.\overline{34} = \frac{133}{99}$$

In $4.1\overline{2}$, the bar is place on one digit so it can be written as $4.1\overline{2} = 4.1222\ldots$ …(i)

by multiplying 10^2 in above equation $4.1\overline{2} \times 10^2 = 412.222\ldots$ …(ii)