ENGINEERING MATHEMATICS

(For ESE & GATE Exam)

(CE, ME, PI, CH, EC, EE, IN, CS, IT)

Salient Features :

- 280 topics under 32 chapters in 8 units
- 640 Solved Examples for comprehensive understanding
- 1479 questions from last 27 years of GATE & ESE exams with detailed solutions
- Only book having complete theory on ESE & GATE Pattern
- Comprising conceptual questions marked with '*' for quick revision

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PREFACE

Science and technology has contributed greatly to the progress of the human race, and thus has made our life quite easier and comfortable. Engineers have played a vital role in the advancements of the technological world. However, with advancements in technology, there come challenges. It is here that the role of Engineering Mathematics comes in play to address the technological challenges posed by the modern era.

Engineering Mathematics is the art of applying mathematics to complicated real-world problems. It is a mix of mathematical theory, practical engineering and scientific computing.

Keeping in mind the importance of Engineering Mathematics skill in modern engineers, in the year 2017, the Union Public Service Commission (UPSC) introduced Engineering Mathematics as a common paper in the syllabus for Engineering Services Examination (ESE), and technical paper for Electrical (EE) stream. It has already been given a weightage of 15% in the Graduate Aptitude Test in Engineering (GATE)

With an objective to develop an ESE or GATE aspirant's numerical abilities and calculation skills, IES Master has come up with this Engineering Mathematics book that brings them face to face with 280 topics under 32 chapters in 8 units, along with previous years questions from GATE (last 27 years), and ESE (last 3 years) and their detailed solutions. Equipped with all this, students can easily decide, how much time to allocate on each chapter based on the number of questions asked in that particular exam.

It is the only book in the market that includes complete theory exactly on the basis of ESE & GATE exams pattern. After each topic, the book includes more than 640 solved examples for concept building & easy learning. To save students time during revision, all the previously asked conceptual questions are marked with '*' symbol.

Our special thanks to the entire IES Master team for their continuous support in bringing out the book. We strongly believe that this book will help students in their journey to success. Suggestions from students, teachers & educators for further improvement in the book are always welcome.

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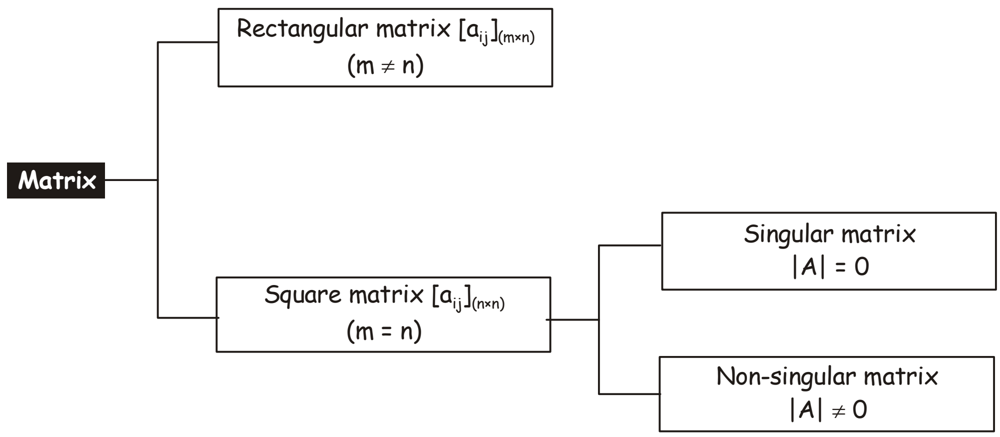
Algebra of Matrices 1.1

DEFINITION OF MATRIX

Matrix is a convenient way of storing information in form of m horizontal rows and n vertical columns.
Matrix can be represented either $A = (a_{ij})_{m \times n}$ or $A = [a_{ij}]_{m \times n}$

TYPES OF MATRICES



Rectangular Matrices:

1. **Row Matrix:** A matrix $A = [a_{ij}]_{m \times n}$ such that $m = 1$ and $n > 1$, matrix is known as row matrix.

   e.g. $A = [a_{ij}]_{1 \times n} = [a_{11}, a_{12}, \ldots a_{1n}]$ or $[a_1, a_2, \ldots, a_n]$ is a row matrix of order n or matrix of order $1 \times n$.

2. **Column Matrix:** A matrix $A = [a_{ij}]_{m \times n}$ such that $m > 1$ and $n = 1$, matrix is known as row matrix.

   e.g. $A = [a_{ij}]_{n \times 1} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ or $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$ is a column matrix of order m or matrix of order $m \times 1$.

3. **Null Matrix or Zero Matrix:** A matrix $A = [a_{ij}]_{m \times n}$ such that $[a_{ij}] = 0$, $\forall$ i, $j \in N$ is called a zero matrix or Null Matrix i.e.

$$O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4. **Horizontal Matrix :** A matrix $A = [a_{ij}]_{m \times n}$ such that $m < n$, matrix is called as horizontal matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}_{3\times 4}$$

5. **Vertical Matrix :** A matrix $A = [a_{ij}]_{m\times n}$ such that m > n, matrix is called as vertical matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}_{4\times 3}$$

Square Matrix:

A matrix in which the number of rows is equal to the number of columns is called a square matrix i.e. $A = (a_{ij})_{m\times n}$ is a square matrix if and only if m = n. A matrix

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$ is a square matrix of order 3. The elements a_{11}, a_{22}, a_{33} of the above square matrix are called its diagonal elements and the diagonal containing these elements is called the principal diagonal or leading diagonal or main diagonal.

Trace of Matrix: The sum of the diagonal elements of a square matrix is called **trace** of the matrix.

1. **Diagonal Matrix:** A square matrix is called diagonal matrix if all its non-diagonal elements are zero i.e. in general a matrix $A = (a_{ij})_{n\times n}$ is called a diagonal matrix if $a_{ij} = 0$ for $i \neq j$;

For example $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a diagonal matrix of order 3′.

2. **Upper Triangular Matrix:** A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ whenever i > j. Thus in an upper triangular matrix all the elements below the principal diagonal are zero. For example,

$$A = \begin{bmatrix} 1 & 2 & 4 & 2 \\ 0 & 3 & -1 & 5 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 4 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

are 4× 4 and 3×3 upper triangular matrices respectively.

3. **Lower Triangular Matrix :** A square matrix of $A = \left[a_{ij}\right]$ is called a lower triangular matrix if $a_{ij} = 0$ whenever i < j. Thus in a lower triangular matrix all the elements above the principal diagonal are zero. For example, $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 4 & 2 & 3 & 0 \\ 5 & 4 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$ are 4×4 and 2×2 lower triangular matrices respectively.

4. **Scalar Matrix:** If all the elements of a diagonal matrix of order n are equal, i.e. if $a_{ii} = k \;\forall\; i$, then the matrix is called a scalar matrix, i.e.

$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix of order 3.

5. **Unit or Identity Matrix:** A square matrix is called a unit matrix or identity matrix if all the diagonal elements are unity and non-diagonal elements are zero. e.g.

PREVIOUS YEARS GATE & ESE QUESTIONS

Questions marked with asterisk (*) are Conceptual Questions

1. The inverse of the matrix $A = \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} \frac{5}{13} & \frac{-1}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{2}{13} & \frac{5}{13} \\ \frac{-1}{13} & \frac{3}{13} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{-1}{13} & \frac{5}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{13} & \frac{-5}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$

[GATE-1994; 1 Mark]

2.* If A and B are real symmetric matrices of order n then which of the following is true.

(a) $AA^T = \mathbf{I}$ (b) $A = A^{-1}$

(c) $AB = BA$ (d) $(AB)^T = B^TA^T$

[GATE-1994 (CS)]

3. The inverse of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ is?

[GATE-1994]

4. The matrix $\begin{bmatrix} 1 & -4 \\ 1 & -5 \end{bmatrix}$ is an inverse of the matrix $\begin{bmatrix} 5 & -4 \\ 1 & -1 \end{bmatrix}$

(a) True (b) False

[GATE-1994 (PI)]

5. The value of the following determinant $\begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$ is

(a) 8 (b) 12

(c) –12 (d) –8

[GATE-1994 (PI)]

6. Given matrix $L = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 5 \end{bmatrix}$ and $M = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ then L×M is

(a) $\begin{bmatrix} 8 & 1 \\ 13 & 2 \\ 22 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 5 \\ 9 & 8 \\ 12 & 13 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 8 \\ 2 & 13 \\ 5 & 22 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & 2 \\ 9 & 4 \\ 0 & 5 \end{bmatrix}$

[GATE-1994 (PI)]

7. The inverse of the matrix $S = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 & -2 \\ -2 & 2 & -2 \\ 0 & 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} \\ 0 & 0 & 1 \end{bmatrix}$

[GATE-1995-EE; 2 Marks]

8.* If A and B are square matrices of size n × n, then which of the following statement is not true.

(a) det(AB) = det(A) det(B)

(b) $\det(kA) = k^n \det(A)$

(c) det(A + B) = det (A) + det (B)

(d) $\det(A^T) = 1/\det(A^{-1})$

[GATE-1995; 1 Mark]

9.* If matrix A is m × n and B is n × p, the number of multiplication operations and addition operations needed to calculate the matrix AB, respectively, are :

(a) mn^2p, mpn (b) mpn, (n – 1)

(c) mpn, mp(n – 1) (d) mn^2p, (m + p)n

[GATE-1995; 1 Mark]

10.* Let A be an invertible matrix and suppose that the inverse of A is $\begin{bmatrix} -1 & 2 \\ 4 & -7 \end{bmatrix}$, the matrix A is

(a) $\begin{bmatrix} 1 & 2/7 \\ 4/7 & 1/7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -4/7 \\ -2/7 & 1/7 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}$

[GATE-1995; 1 Mark]

11.* The matrix B = A^T, where A is any matrix is

(a) skew symmetric

(b) symmetric about the secondary diagonal

(c) always symmetric

(d) another general matrix.

[GATE-1996; 1 Mark]

12. If $A^T = A^{-1}$, where A is a real matrix, then A is

(a) normal (b) symmetric

(c) Hermitian (d) orthogonal

[GATE-1996; 1 Mark]

13.* If A and B are non-zero square matrices, then AB = 0 implies

(a) A and B are orthogonal

(b) A and B are singular

(c) B is singular

(d) A is singular

[GATE-1996; 1 Mark]

14.* The matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under multiplication.

(a) If a = b or $\theta = n\pi, n$ is an integer

(b) always

(c) never

(d) If $a\cos\theta \neq b\sin\theta$

[GATE-1996 (CS)]

15.* Let A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and B = $\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be two matrices such that AB = I. Let C = A$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and CD = I. Express the elements of D in terms of the elements of B.

[GATE-1996 (ME)]

16.* Let $A_{n\times n}$ be matrix of order n and I_{12} be the matrix obtaind by interchanging the first and second rows of I_n. Then AI_{12} is such that its first.

(a) row is the same as its second row

(b) row is the same as second row of A

(c) column is the same as the second column of A

(d) row is a zero row.

[GATE-1997 (CS)]

17.* The determinant of the matrix $\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

(a) 11 (b) –48

(c) 0 (d) –24

[GATE-1997 (ME)]

18.* In A and B are two matrices and if AB exists, then BA exists

(a) only if A has as many row as B has columns

(b) only if both A and B are square matrices

(c) only if A and B are skew symmetric matrices

(d) only if both A and B are symmetric.

[GATE-1997-CE; 1 Mark]

19. Inverse of matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

[GATE-1997-CE; 1 Mark]

20.* If the determinant of matrix $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -6 \\ 2 & 7 & 8 \end{bmatrix}$ is 26, then the determinant of the matrix $\begin{bmatrix} 2 & 7 & 8 \\ 0 & 5 & -6 \\ 1 & 3 & 2 \end{bmatrix}$ is

(a) –26 (b) 26

(c) 0 (d) 52

[GATE-1997-CE; 1 Mark]

21. If A = $\begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ then A^{-1} =

SOLUTIONS

Sol–1: (c)

$$A = \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -3 & 5 \\ 2 & 1 \end{vmatrix} = -3 - 10$$

$$= -13 \neq 0 \quad \Rightarrow \quad A^{-1} \text{ exists}$$

Now $\quad \text{Cof}(A) = \begin{bmatrix} 1 & -2 \\ -5 & -3 \end{bmatrix}$

so, $\quad = \begin{bmatrix} 1 & -2 \\ -5 & -3 \end{bmatrix}^T = \begin{bmatrix} 1 & -5 \\ -2 & -3 \end{bmatrix}$

Hence, $\quad A^{-1} = \dfrac{\text{adj}A}{|A|} = -\dfrac{1}{13}\begin{bmatrix} 1 & -5 \\ -2 & -3 \end{bmatrix}$

$$= \begin{bmatrix} -\frac{1}{13} & \frac{5}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$$

Sol–2: (d)

$(AB)^T = B^T A^T$ by using Reversal Law.

Sol–3:

$$\text{Cof}(A) = \begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & -1 \\ -1 & -2 & 1 \end{bmatrix}$$

& $|A| = 1(0-1+0+1(-1-0) = -2 \neq 0$ so A^{-1} exist.

So $\quad \text{adj} = (\text{Cof } A)^T = \begin{bmatrix} -1 & 1 & -1 \\ 0 & 0 & -2 \\ -1 & -1 & 1 \end{bmatrix}$

$\therefore \quad A^{-1} = \dfrac{\text{adj}(A)}{|A|} = \dfrac{1}{(-2)}\begin{bmatrix} -1 & 1 & -1 \\ 0 & 0 & -2 \\ -1 & -1 & 1 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

Sol–4: (a)

$$AA^{-1} = I$$

$$\begin{bmatrix} 5 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 5-4 & -20+20 \\ 1-1 & -4+5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, the given statement is frue.

Sol–5: (d)

$$|A| = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

$$= 1(225-256) - 4(100-144) + 9(64-81)$$

$$= -8$$

Sol–6: (b)

$$LM = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 5 \end{bmatrix}_{3\times 2}\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 6 & 5 \\ 9 & 8 \\ 12 & 13 \end{bmatrix}_{3\times 2}$$

Sol–7: (d)

$$S = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Determinant of matrix S

$$|S| = 1(1-0) + 1(1-0)$$

$$= 2 \neq 0 \quad \Rightarrow \quad S^{-1} \text{ exist.}$$

Now, $\quad \text{cof}(S) = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$

$\Rightarrow \quad \text{adj}(S) = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}^T$

$\therefore \quad \text{adj}(S) = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$

$\Rightarrow \quad [S]^{-1} = \dfrac{\text{adj}(S)}{|S|} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 \end{bmatrix}$

Sol–8: (c)

(a) $\quad |AB| = |A|\,|B|$

It is true

(b) $\quad |kA| = k^n|A|$

where n is order of matrix

(c) $\quad |A + B| = |A| + |B|$

It is false

(d) $\quad |A^T| = |A| = \dfrac{1}{|A^{-1}|}$

which is true. Hence (c) is the correct answer.