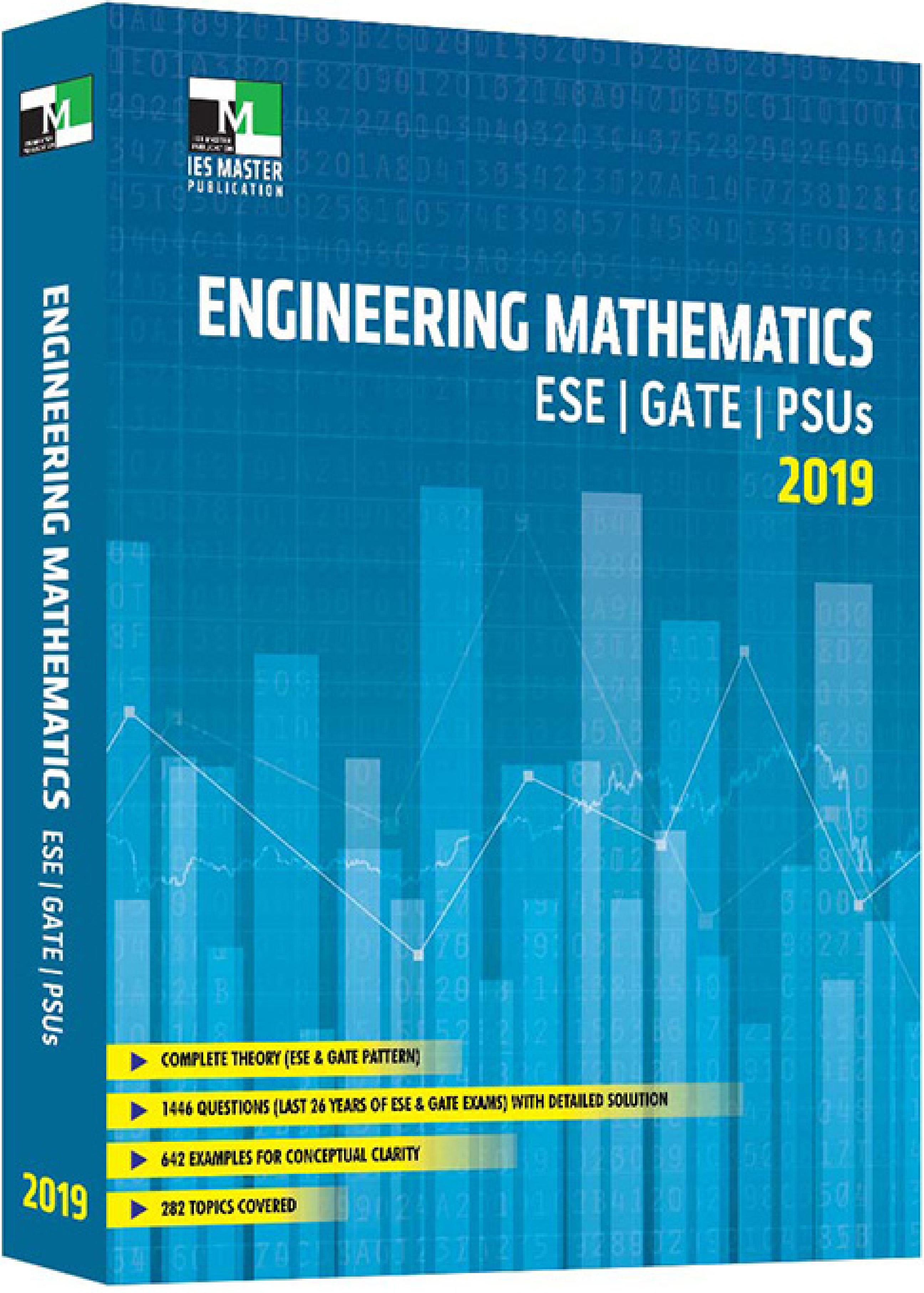
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COMPLETE THEORY (ESE & GATE PATTERN)
1446 QUESTIONS (LAST 26 YEARS OF ESE & GATE EXAMS) WITH DETAILED SOLUTION
642 EXAMPLES FOR CONCEPTUAL CLARITY
282 TOPICS COVERED
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PREFACE

We, at IES MASTER, have immense pleasure in placing the second edition of **“Engineering Mathematics”** before the aspirants of GATE & ESE exams.

Dear Students, as we all know that in 2016 UPSC included Engineering Mathematics as a part of syllabus of common paper for ESE exam as well as of a technical paper for EC/EE branch, while Engineering Mathematics already has 15% weightage in GATE exam. We have observed that currently available books cover neither all the topics nor all previously asked questions in GATE & ESE exams. Since most of the books focus on only some selected main topics, students have not been able to answer more than 60-65% of 1446 questions that have been asked in GATE & ESE exams so far. Hence to overcome this problem, we have tried our best by covering more than 282 topics under 32 chapters in 8 units. (One should not be in dilemma that 282 topics are more than sufficient. These are the minimum topics from where GATE & ESE have already asked questions). Since we have covered every previous year questions from last 26 years of each topic, students can easily decide, how much time to allocate on each chapter based on the number of questions asked in that particular exam. Again, we have included only those proofs that are necessary for concept building of topics and we have stressed on providing elaborate solution to all the questions.

It is the only book in the market that includes complete theory exactly on ESE & GATE Pattern. After each topic there are sufficient number of solved examples for concept building & easy learning. The book includes such types of 642 examples. It also covers all the previously asked questions in which conceptual questions are marked with ‘*’ sign so that students can save their time, while revising.

Having incorporated my teaching experience of more than 14 years, I believe this book will enable the students to excel in Engineering Mathematics.

My source of inspiration is Mr. Kanchan Thakur Sir (Ex-IES). He has continuously motivated me while writing this book.

My special thanks to the entire IES MASTER Team for their continuous support in bringing out the book. I strongly believe that this book will help students in their journey to success. I invite suggestions from students, teachers & educators for further improvement in the book.

Dr. Puneet Sharma
(M.Sc., Ph.D.)
IES Master Publications
New Delhi

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UNIT 4 : COMPLEX ANALYSIS

UNIT 6 : NUMERICAL METHODS

UNIT 7 : TRANSFORM THEORY

Algebra of Matrices

1.1

DEFINITION OF MATRIX

A set of m n objects or numbers (real or complex) arranged in a rectangular array of m rows and n columns, i.e.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{m_1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

is called a matrix of order m × n matrix.

The number a_{11}, a_{12}, ..., a_{mn} are called the elements of the matrix. The vertical lines are columns or column vectors, and the horizontal lines are called rows or row vectors of the matrix.

TYPES OF MATRICES

1. **Row Matrix:** A matrix having one row and any number of columns is called a row matrix, or a row vector, e.g., $[a_{11}, a_{12}, ... a_{1n}]$ or $[a_1, a_2, ..., a_n]$ is a row matrix of order n or matrix of order 1 × n.
2. **Column Matrix:** A matrix having one column and any number of rows is called a column matrix or a column vector. e.g. $\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ or $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$ is a column matrix of order m or matrix of order m × 1.
3. **Null Matrix or Zero Matrix:** Any matrix in which all the elements are zero is called a zero matrix or Null Matrix i.e.

$$O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4. **Square Matrix:** A matrix in which the number of rows is equal to the number of columns is called a square matrix i.e. $A = (a_{ij})_{m\times n}$ is a square matrix if and only if m = n. A matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

is a square matrix of order 3. The elements a_{11}, a_{22}, a_{33} of the above square matrix are called its diagonal elements and the diagonal containing these elements is called the principal diagonal or leading diagonal or main diagonal.

Trace of Matrix: The sum of the diagonal elements of a square matrix is called **trace** of the matrix.

5. Diagonal Matrix: A square matrix is called diagonal matrix if all its non-diagonal elements are zero i.e. in general a matrix $A = (a_{ij})_{n\times n}$ is called a diagonal matrix if $a_{ij} = 0$ for $i \neq j$;

For example $$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$ is a diagonal matrix of order 3´.

6. Scalar Matrix: If all the elements of a diagonal matrix of order n are equal, i.e. if $a_{ii} = k \, \forall \, i$, then the matrix is called a scalar matrix, i.e.

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$ is a scalar matrix of order 3.

7. Unit or Identity Matrix: A square matrix is called a unit matrix or identity matrix if all the diagonal elements are unity and non-diagonal elements are zero. e.g.

$$I_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, I_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

are identity matrices of order 3×3 and 2×2 respectively.

8. Upper Triangular Matrix: A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ whenever $i > j$. Thus in an upper triangular matrix all the elements below the principal diagonal are zero. For example,

$$A = \begin{bmatrix} 1 & 2 & 4 & 2 \\ 0 & 3 & -1 & 5 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 4 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

are 4× 4 and 3×3 upper triangular matrices respectively.

9. Lower Triangular Matrix : A square matrix of $A = \left[a_{ij}\right]$ is called a lower triangular matrix if $a_{ij} = 0$ whenever $i < j$. Thus in a lower triangular matrix all the elements above the principal diagonal are zero. For example, $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 4 & 2 & 3 & 0 \\ 5 & 4 & 3 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$ are 4×4 and 2×2 lower triangular matrices respectively.

10. Sub matrix : A matrix obtained from a given matrix, say $A = (a_{ij})_{m\times n}$ by deleting some rows or columns or both is called a sub matrix of A. For example if $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 5 & 1 \\ 7 & 8 & 0 & 2 \\ 1 & 7 & 2 & 3 \end{bmatrix}$ then the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 7 & 8 & 0 \end{bmatrix}$ and $\begin{bmatrix} 3 & 5 \\ 8 & 0 \\ 7 & 2 \end{bmatrix}$ are sub matrices of A.

11. Equal matrices :

Two matrices are said to be equal if :

(i) They are of the same order.

(ii) The elements in the corresponding positions are equal.

Thus if $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$. Then A = B

In general if $A = \left(a_{ij}\right)_{m\times n}$ and $B = \left(b_{ij}\right)_{m\times n}$ are matrices each of order m×n and $a_{ij} = b_{ij}$ for all i and j then A = B.

PRODUCT OF MATRIX BY A SCALAR (OR CONSTANT)

Let $A = \left[a_{ij}\right]_{m\times n}$ be a matrix of order m×n and k is a constant, then their product is matrix $kA = \left[ka_{ij}\right]_{m\times n}$ i.e. every element of A is multiplied by k. For example, if $A = \begin{bmatrix} 2 & -1 & 0 \\ 4 & 5 & -3 \end{bmatrix}$. Then we have $4A = \begin{bmatrix} 8 & -4 & 0 \\ 16 & 20 & -12 \end{bmatrix}$

ADDITION AND SUBTRACTION OF MATRICES

Let A and B be two matrices of the same order, then their sum A + B is defined as the matrix each element of which is the sum of the corresponding elements of A and B.

In general if $A = \left(a_{ij}\right)_{m\times n}$ and $B = \left(b_{ij}\right)_{m\times n}$ then their sum is defined by the matrix.

$$C = A + B = (C_{ij})_{m\times n}$$

where $c_{ij} = a_{ij} + b_{ij}$ and i = 1, 2, ...m and j = 1, 2, 3,.......,n

If $A = \begin{bmatrix} 4 & 2 & 5 \\ 11 & 13 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$

Then
$$A + B = \begin{bmatrix} 4+1 & 2+0 & 5+2 \\ 11+3 & 13+1 & -6+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 7 \\ 14 & 14 & -2 \end{bmatrix}$$

Similarly, if A and B are two matrices of the same order, then their difference is defined by

$$A - B = \begin{bmatrix} 4-1 & 2-0 & 5-2 \\ 11-3 & 13-1 & -6-4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ 8 & 12 & -10 \end{bmatrix}$$

Properties of Matrix Addition

(i) Matrix addition is commutative :

Let $A = \left[a_{ij}\right]_{m\times n}$ and $B = \left[b_{ij}\right]_{m\times n}$ be matrices of the same order m×n then A + B = B + A.

(ii) Matrix addition is associative :

Let A, B, C can be the matrices of the same order, Then $(A+B)+C = A+(B+C)$.

(iii) Cancellation law for matrix addition :

Let A, B, C be the matrices of the same order, then

A + B = A + C holds if and only if B = C.

MULTIPLICATION OF MATRICES

The product AB of two matrices A and B is possible only when the number of columns in A is equal to the number of rows in B. Such matrices are said to be conformable for multiplication.

Let $A = \left[a_{ij}\right]_{m\times n}$ be a matrix of order m×n and $B = \left[b_{jk}\right]_{n\times p}$ be a matrix of order n×p, then the product AB is defined as a matrix $C = \left[c_{ik}\right]_{m\times p}$ of order m×p

where
$$c_{ik} = \sum_{j=1}^{m} a_{ij}b_{jk} = a_{il}b_{lk} + a_{i2}b_{2k} + ... + a_{in}b_{nk}$$

i.e. (i, k)th element of AB = sum of the products of the elements of ith row of A with the corresponding elements of kth column of B, i.e.

Example 1 : If $A=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$, $B=\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$. Show that $AB \neq BA$

Solution : $AB = \begin{bmatrix}1.2-2.4+3.2 & 1.3-2.5+3.1\\ -4.2+2.4+5.2 & -4.3+2.5+5.1\end{bmatrix}=\begin{bmatrix}0 & -4\\ 10 & 3\end{bmatrix}_{2\times 2}$

Since number of columns in B is equal to number of rows in A, so BA is also defined.

$$BA = \begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}=\begin{bmatrix}-10 & 2 & 27\\ -16 & 2 & 27\\ -2 & -2 & 11\end{bmatrix}$$

Hence $AB \neq BA$.

Thus matrix multiplication is not commutative in general.

Example 2 : If $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 2 & 1\\ 2 & 3 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 2\\ 3 & 0\\ 4 & 1\end{bmatrix}$. Find AB. Will BA exist?

Solution : $AB = \begin{bmatrix}0 & 1 & 0\\ 0 & 2 & 1\\ 2 & 3 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 0\\ 4 & 1\end{bmatrix}=\begin{bmatrix}3 & 0\\ 10 & 1\\ 15 & 5\end{bmatrix}$

Now, since number of columns of B is 2 and number of rows in A are 3. So BA does not exist. Therefore if AB is defined, it is not at all necessary that the product BA is also defined.

Properties of Matrix Multiplication

1. Multiplication of matrices is not commutative i.e. $AB \neq BA$
2. Multiplication of matrices is associative, i.e. A(BC) = (AB)C
3. Matrix multiplication is distributive with respect to addition i.e. A(B + C) = AB + AC

   where A, B C are any three matrices of order $m\times n$, $n\times p$, $n\times p$ respectively.
4. **Positive integral power of square matrix :**

   The product of AA is defined only when A is square matrix of order n. We shall denote it as A^2. If m and n are any positive integers, then we have $A^m A^n = A^{m+n}$
5. **Zero Divisor :**

   AB = O does not necessarily imply that at least one of the matrices A and B must be zero matrix i.e. the product of two matrices can be zero matrix while neither of them is a zero matrix. Such matrices are said to be zero divisor. For example; if

   $A = \begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix}$, then $AB=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

   i.e. neither the matrix A nor matrix B is a zero matrix but their product matrix AB is zero matrix.

6. Multiplication with Identity Matrix :

If A be $n\times n$ matrix and I_n is a unit matrix of order n, then $AI_n = I_nA = A$

i.e. a matrix remains unaltered when it is multiplied by a unit matrix of same order.

MINORS OF MATRIX

The determinant value of the square matrix obtained from the original matrix of any order by the omission of the rows and columns is called a minor of a matrix. For example

If $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 7 & 1 \\ 8 & 2 & 7 & 3 \end{bmatrix}_{3\times 4}$ is a matrix of order 3 × 4. Then minors of A are $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 3 & 7 \\ 8 & 2 & 7 \end{vmatrix}; \begin{vmatrix} 0 & 3 \\ 8 & 2 \end{vmatrix}$ & $\begin{vmatrix} 7 & 1 \\ 7 & 3 \end{vmatrix}$ etc.

COFACTORS OF A MATRIX

The cofactors of the element a_{ij} is defined as

$$A_{ij} = (-1)^{i+j}|M_{ij}|$$

where $|M_{ij}|$ is the determinant obtained by deleting the i^{th} row and j^{th} column from given matrix.

EXPANSION BY COFACTORS

We have row expansion of a determinant of a general 3×3 matrix, that is

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Since $\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = M_{11}$, $\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = M_{12}$, and $\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = M_{13}$. So we have,

$$|A| = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13} = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

Since by the definition of cofactors, we have

$$A_{11} = (-1)^{1+1}M_{11} = M_{11},$$

$$A_{12} = (-1)^{1+2}M_{12} = -M_{12},$$

and

$$A_{13} = (-1)^{1+3}M_{13} = M_{13}.$$

PROPERTIES OF DETERMINANTS

1. $|A'| = |A|$
2. If we interchange any two rows or columns then sign of determinants change.
3. If any 2 rows or any 2 columns in a determinant are identical (or proportional) then the value of the determinant is zero.
4. Multiplying a determinant by K means multiplying the elements of only one row (or one column) by K.

   e.g. $$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2$$

   then $$2|A| = 2\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 4 \\ 3 & 4 \end{vmatrix} = 8 - 12 = -4 = 2(-2)$$

5. If elements of a row in a determinant can be expressed as the sum of two or more elements then the given determinant can be expressed as the sum of 2 or more determinants.

   e.g. $$|A| = \begin{vmatrix} a+c & b+d \\ e & f \end{vmatrix} = \begin{vmatrix} a & b \\ e & f \end{vmatrix} + \begin{vmatrix} c & d \\ e & f \end{vmatrix}$$

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