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ENGINEERING MATHEMATICS

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2018

- COMPLETE THEORY (ESE & GATE PATTERN)
- 1386 QUESTIONS (LAST 25 YEARS OF ESE & GATE EXAMS) WITH DETAILED SOLUTION
- 672 EXAMPLES FOR CONCEPTUAL CLARITY
- 289 TOPICS COVERED

ENGINEERING MATHEMATICS

(For ESE & GATE Exam)

(CE, ME, PI, CH, EC, EE, IN, CS, IT)

Salient Features :

- **289 topics under 31 chapters in 8 units**
- **672 Solved Examples for comprehensive understanding**
- **1386 questions from last 25 years of GATE & ESE exams with detailed solutions**
- **Only book having complete theory on ESE & GATE Pattern**
- **Comprising conceptual questions marked with '*' to save the time while revising**

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PREFACE

We, the IES MASTER, have immense pleasure in placing the first edition of **"Engineering Mathematics"** before the aspirants of GATE & ESE exams.

Dear Students, as we all know that in 2016 UPSC included Engineering Mathematics as a part of syllabus of common paper for ESE exam as well as of a technical paper for EC/EE branch, while Engineering Mathematics already has 15% weightage in GATE exam. We have observed that currently available books cover neither all the topics nor all previously asked questions in GATE & ESE exams. Since most of the books focus on only some selected main topics, students have not been able to answer more than 60-65% of 1386 questions that have been asked in GATE & ESE exams so far. Hence to overcome this problem, we have tried our best by covering more than 289 topics under 31 chapters in 8 units. (One should not be in dilemma that 289 topics are more than sufficient. These are the minimum topics from where GATE & ESE have already asked questions). Since we have covered every previous year questions from last 25 years of each topic, students can easily decide, how much time to allocate on each chapter based on the number of questions asked in that particular exam. Again, we have included only those proofs that are necessary for concept building of topics and we have stressed on providing elaborate solution to all the questions.

It is the only book in the market which has complete theory exactly on ESE & GATE Pattern. After each topic there are sufficient number of solved examples for concept building & easy learning. The book includes such types of 672 examples. It also covers all the previously asked questions in which conceptual questions are marked with '*' sign so that students can save their time, while revising.

Having incorporated my teaching experience of more than 13 years, I believe this book will enable the students to excel in Engineering Mathematics.

My source of inspiration is Mr. Kanchan Thakur Sir (Ex-IES). He has continuously motivated me while writing this book.

My special thanks to the entire IES MASTER Team for their continuous support in bringing out the book. I strongly believe that this book will help students in their journey of success. I invite suggestions from students, teachers & educators for further improvement in the book.

Dr. Puneet Sharma
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IES Master Publications
New Delhi

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Institute for Engineers (IES/GATE/PSUs)

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SESSION 2018-2019
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12th Feb / 26th Feb — **Regular Batch** (CE/ME/EE/ECE)

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27th Jan / 17th Feb — **Weekend Batch** (CE/ME/EE/ECE)

CONTENTS

UNIT 2 : VECTOR CALCULUS

UNIT 3 : COMPLEX ANALYSIS

UNIT 5 : NUMERICAL METHODS

UNIT 7 : LINEAR ALGEBRA

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PREVIOUS YEARS GATE & ESE QUESTIONS

Questions marked with aesterisk (*) are Conceptual Questions

1.* $\lim\limits_{x\to 0}\dfrac{x(e^x-1)+2(\cos x-1)}{x(1-\cos x)}$ = ______

[GATE-1993 (ME)]

2. $\lim\limits_{x\to 0}\dfrac{1}{10}\dfrac{1-e^{-j5x}}{1-e^{-jx}}$ = ______

(a) 0 (b) 1.1
(c) 0.5 (d) 1

[GATE-1993]

3. The value of $\left[\lim\limits_{x\to\infty}\dfrac{\sin x}{x}\right]$ is:

(a) ∞ (b) 2
(c) 1 (d) 0

[GATE-1994; 1 Mark]

4. The value of $\left[\lim\limits_{x\to\infty}\left(\dfrac{1}{\sin x}-\dfrac{1}{\tan x}\right)\right]$ is :

(a) 0 (b) 1
(c) 2 (d) ∞

[GATE-1994; 1 Mark]

5. The limit of $\underset{\theta\to 0}{\text{Lt}}\dfrac{\sin\theta}{\theta}$ is

(a) 1
(b) 0
(c) ∞
(d) None of the above

[GATE-1994; 1 Mark]

6.* $\lim\limits_{x\to\infty}\dfrac{x^3-\cos x}{x^2+(\sin x)^2}$ equal

(a) ∞ (b) 0
(c) 2 (d) does not exist

[GATE-1995-CS; 2 Marks]

7.* The function f(x) = |x + 1| on the interval [–2, 0] is

(a) continuous and differentiable
(b) continuous on the integers but not differentiable at all points
(c) neither continuous nor differentiable
(d) differentiable but not continuous

[GATE-1995; 1 Mark]

8. $\lim\limits_{x\to 0} x\sin\left(\dfrac{1}{x}\right)$ is

(a) ∞ (b) 0
(c) 1 (d) Non-existant

[GATE-1995; 1 Mark]

9. If a function is continuous at a point its first derivative

(a) may or may not exist
(b) exists always
(c) will not exist
(d) has unique value

[GATE-1996; 1 Mark]

10. If y = |x| for x < 0 and y = x for $x\geq 0$, then

(a) $\dfrac{dy}{dx}$ is discontinuous at x = 0
(b) y is discontinuous at x = 0
(c) y is not defined at x = 0
(d) None of these

[GATE-1997; 1 Mark]

11. $\lim\limits_{\theta\to 0}\dfrac{\sin m\theta}{\theta}$, where m is an integer, is one of the following:

(a) m (b) $m\pi$
(c) $m\theta$ (d) 1

[GATE-1997; 1 Mark]

12.* Find the limiting value of the ratio of the square of the sum of a natural numbers to n times the sum of squares of the n natural number as, n approaches infinity.

[GATE-1997]

13.* The profile of a cam in a particular zone is given by x = $\sqrt{3}\cos\theta$ and y = $\sin\theta$. The normal to the cam profile at $\theta=\dfrac{\pi}{4}$ is at an angle (with respect to x axis):

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{3}$ (d) 0

[GATE-1998]

14. Limit of the function $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}}$ is

(a) $\frac{1}{2}$ (b) 0

(c) ∞ (d) 1

[GATE-1999; 1 Mark]

15. Value of the function $\lim_{x\to a} (x-a)^{(x-a)}$ is

(a) 1 (b) 0

(c) ∞ (d) a

[GATE-1999; 1 Mark]

16. $\underset{x\to 1}{\text{Lt}} \frac{(x^2-1)}{(x-1)}$ is

(a) ∞ (b) 0

(c) 2 (d) 1

[GATE-2000; 1 Mark]

17. The limit of the function f(x) = $\left[1-\frac{a^4}{x^4}\right]$ as $x\to\infty$ is given by

(a) 1 (b) $\exp[-a^4]$

(c) ∞ (d) Zero

[GATE-2000; 1 Mark]

18. What is the derivative of f(x) = |x| at x = 0?

(a) 1 (b) –1

(c) 0 (d) Does not exist

[GATE-2001; 1 Mark]

19. $\lim_{x\to\frac{\pi}{4}} \frac{\sin 2\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}} =$ ______

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

[GATE-2001 (IN)]

20.* Which of the following functions is not differentiable in the domain [–1,1]?

(a) $f(x) = x^2$

(b) f(x) = x – 1

(c) f(x) = 2

(d) f(x) = maximum(x, –x)

[GATE-2002; 1 Mark]

21.* The limit of the following sequence as $n\to\infty$ is:

$$x_n = n^{1/n}$$

(a) 0 (b) 1

(c) ∞ (d) $-\infty$

[GATE-2002-CE; 1 Mark]

22. $\lim_{x\to 0} \frac{\sin^2 x}{x}$ is equal to

(a) 0 (b) ∞

(c) 1 (d) –1

[GATE-2003; 1 Mark]

23. The value of the function f(x) = $\lim_{x\to 0} \frac{x^3+x^2}{2x^3-7x^2}$ is

(a) zero (b) $-\frac{1}{7}$

(c) $\frac{1}{7}$ (d) infinite

[GATE-2004; 1 Mark]

24. If $x = a(\theta+\sin\theta)$ and $y = a(1-\cos\theta)$, then dy/dx will be equal to

(a) $\sin\left(\frac{\theta}{2}\right)$ (b) $\cos\left(\frac{\theta}{2}\right)$

(c) $\tan\left(\frac{\theta}{2}\right)$ (d) $\cot\left(\frac{\theta}{2}\right)$

[GATE-2004]

25.* With a 1 unit change in b, what is the change in x in the solution of the system of equations x + y = 2, 1.01x + 0.99 y = b?

(a) zero (b) 2 units

(c) 50 units (d) 100 units

[GATE-2005]

26.* Equation of the line normal to function f(x) = $(x-8)^{2/3}+1$ at P(0, 5) is

(a) y = 3x – 5 (b) y = 3x + 5

(c) 3y = x + 15 (d) 3y = x – 5

[GATE-2006; 2 Marks]

27. If f(x) = $\frac{2x^2-7x+3}{5x^2-12x-9}$, then $\lim_{x\to 3}$ f(x) is

(a) – 1/3 (b) 5/18

(c) 0 (d) 2/5

[GATE-2006; 2 Marks]

28.* If, $y = x+\sqrt{x+\sqrt{x+\sqrt{x+...\infty}}}$, then y(2) =

ANSWER KEY

1.	**(See Sol.)**	**21.**	**(b)**	**41.**	**(b)**	**61.**	**(c)**
2.	**(c)**	**22.**	**(a)**	**42.**	**(d)**	**62.**	**(c)**
3.	**(d)**	**23.**	**(b)**	**43.**	**(c)**	**63.**	**(c)**
4.	**(d)**	**24.**	**(c)**	**44.**	**(c)**	**64.**	**(c)**
5.	**(a)**	**25.**	**(c)**	**45.**	**(b)**	**65.**	**(c)**
6.	**(a)**	**26.**	**(b)**	**46.**	**(b)**	**66.**	**(0.5)**
7.	**(b)**	**27.**	**(b)**	**47.**	**(a)**	**67.**	**(1)**
8.	**(b)**	**28.**	**(b)**	**48.**	**(b)**	**68.**	**(d)**
9.	**(a)**	**29.**	**(b)**	**49.**	**(d)**	**69.**	**(25)**
10.	**(a)**	**30.**	**(a)**	**50.**	**(a)**	**70.**	**(b)**
11.	**(a)**	**31.**	**(c)**	**51.**	**(c)**	**71.**	**(d)**
12.	**(0.75)**	**32.**	**(c)**	**52.**	**(c)**	**72.**	**(b)**
13.	**(c)**	**33.**	**(b)**	**53.**	**(a)**	**73.**	**(a)**
14.	**(d)**	**34.**	**(a)**	**54.**	**(c)**	**74.**	**(d)**
15.	**(a)**	**35.**	**(b)**	**55.**	**(–2)**	**75.**	**(c)**
16.	**(c)**	**36.**	**(c)**	**56.**	**(c)**	**76.**	**(1)**
17.	**(a)**	**37.**	**(c)**	**57.**	**(–0.33)**		
18.	**(d)**	**38.**	**(c)**	**58.**	**(d)**		
19.	**(d)**	**39.**	**(a)**	**59.**	**(a)**		
20.	**(d)**	**40.**	**(c)**	**60.**	**(c)**		

SOLUTIONS

Sol–1:

Using L' Hospital Rule

$$\text{Req. Limit} = \lim_{x\to 0}\frac{(e^x-1)+xe^2-2\sin x}{(1-\cos x)+x(\sin x)}\left(\frac{0}{0}\right)$$

Again using L' Hospital rule

$$= \lim_{x\to 0}\frac{(e^x+e^x+xe^2-2\cos x}{\sin x+\sin x+x\cos x}\left(\frac{0}{0}\right)$$

Again using L' Hospital Rule

$$= \lim_{x\to 0}\frac{e^x+e^x+e^x+xe^x+2\sin x}{\cos x+\cos x+\cos x-x\sin x}$$

$$= \frac{1+1+1+0+0}{1+1+1-0}=1$$

Sol–2: (c)

Using L' Hospital rule

$$\lim_{x\to 0}\cdot\frac{1-e^{-j5x}}{10(1-e^{-jx})} = \lim_{x\to 0}\frac{1}{10}\times\frac{5je^{-j5x}}{je^{-jx}}=\frac{5}{10}=0.5$$

Sol–3: (d)

$$\lim_{x\to\infty}\frac{\sin x}{x} = \lim_{y\to 0}\frac{\sin 1/y}{1/y}$$

$$= \lim_{y\to 0} y\sin\left(\frac{1}{y}\right)=0$$

$$\boxed{\lim_{x\to\infty}\frac{\sin x}{x}=0} \qquad (\because\ |\sin x| \le 1)$$

Sol–4: (d)

$$\lim_{x\to\infty}\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right) = \lim_{x\to\infty}\left(\frac{1-\cos x}{\sin x}\right)$$

$$= \lim_{x\to\infty}\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \lim_{x\to\infty}\tan\frac{x}{2} = \infty$$

Sol–5: (a)

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

Sol–6: (a)

$$\lim_{x\to\infty}\frac{x^3-\cos x}{x^2+(\sin x)^2} = \lim_{x\to\infty}\frac{1-\frac{\cos x}{x^3}}{\frac{x^2}{x^3}+\frac{(\sin x)^2}{x^3}}$$

$$= \lim_{x\to\infty}\frac{1-\frac{\cos x}{x^3}}{\frac{1}{x}+\frac{(\sin x)^2}{x^3}}$$

$$= \frac{1-0}{\frac{1}{\infty}+0} = \frac{1}{0}$$

$$\boxed{\lim_{x\to\infty}\frac{x^3-\cos x}{x^2+(\sin x)^2} = \infty}$$

Sol–7: (b)

$$f(x) = |x+1|$$
$$= -(x+1) \quad \text{for } x < -1$$
$$= (x+1) \quad \text{for } x \geq -1$$

Only concern is $x = -1$

$$\text{Left limit} = \lim_{x\to -1^-} -(x+1) = 0$$

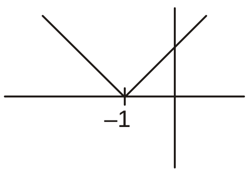
$$\text{Right limit} = \lim_{x\to -1^+} (x+1) = 0$$

$$\boxed{f(x) \text{ is continuous in the Interval } [-2, 0]}$$

$$\text{LD} = -1$$
$$\text{Right Derivative} = +1$$

$$\boxed{\text{Hence } f(x) \text{ is not differentiable at } x = -1}$$

Alternative Solution :



It is clear from graph f(x) is continuous every where but not differentiate at x = –1 because there is sharp change in slope at x = –1.

Sol–8: (b)

$$\lim_{x\to 0} x\sin\frac{1}{x} = 0\cdot\sin\infty$$
$$= 0 \times [-1, 1]$$
$$= 0$$

Sol–9: (a)

If f (x) is continuous at a point x = a then it may or may not be differentiable at x = a.

Sol–10: (a)

$$y = |x|$$

$$= \begin{cases}-x, & \text{for } x < 0\\ x, & \text{for } x > 0\end{cases} \quad \text{and} \quad \frac{dy}{dx} = \begin{cases}-1, & x < 0\\ 1, & x > 0\end{cases}$$

$$\text{left limit} = \lim_{x\to 0}(-x) = 0$$

$$\text{Right limit} = \lim_{x\to 0}(x) = 0$$

$$\because \quad \text{Left limit} = \text{Right limit}$$

$$\boxed{y \text{ is continuous at } x = 0}$$

$$\text{Left derivative} = -1$$
$$\text{Right derivative} = +1$$
$$\because \quad \text{Left derivative} \neq \text{Right derivative}$$

$$\boxed{\frac{dy}{dx} \text{ is discontinuous at } x = 0}$$

Sol–11: (a)

$$\lim_{\theta\to 0}\frac{\sin m\theta}{\theta}\left(\frac{0}{0}\right)$$

Apply LH Rule

$$\lim_{\theta\to 0}\frac{m\cos m\theta}{1} = m$$

Alternative Solution:

$$\lim_{\theta\to 0}\frac{\sin m\theta}{\theta} = \lim_{\theta\to 0} m\frac{\sin m\theta}{m\theta} = m(1) = m$$

Sol–12: (0.75)

Sum of the squares of natural numbers = S_2

$$S_2 = 1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + n^2$$

$$S_2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of natural numbers = S_1

$$S_1 = 1 + 2 + 3 + 4 + \ldots\ldots + n$$

$$S_1 = \frac{n(n+1)}{2}$$

Hence,

$$\Rightarrow \quad \lim_{n\to\infty}\frac{\left[\frac{n(n+1)}{2}\right]^2}{n\times\left[\frac{n(n+1)(2n+1)}{6}\right]}$$

$$\Rightarrow \lim_{n\to\infty} \frac{\frac{n^2(n+1)^2}{4}}{\frac{n\times n(n+1)(2n+1)}{6}}$$

$$\Rightarrow \lim_{n\to\infty} \frac{3}{2}\frac{(n+1)}{(2n+1)}$$

$$\Rightarrow \lim_{n\to\infty} \frac{3}{2}\left(\frac{1+\frac{1}{n}}{2+\frac{1}{n}}\right)$$

$$\Rightarrow \frac{3}{4} = 0.75$$

Sol–13: (c)

$$x = \sqrt{3}\cos\theta$$
$$y = \sin\theta$$

Slope of cam profile at an angle θ is

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{1}{\left(\frac{dx}{d\theta}\right)}$$

$$= \frac{\cos\theta}{-\sqrt{3}\sin\theta} = \frac{-\cot\theta}{\sqrt{3}}$$

Slope of Normal to cam profile at angle θ is

$$m = \frac{-1}{\left(\frac{dy}{dx}\right)} = \sqrt{3}\tan\theta$$

$$m = \sqrt{3}\tan\left(\frac{\pi}{4}\right)$$

$$m = \sqrt{3}$$

Angle of Normal with x-axis is

$$\alpha = \tan^{-1}(m) = \tan^{-1}\left(\sqrt{3}\right)$$

$$\boxed{\alpha = 60^\circ = \frac{\pi}{3}}$$

Sol–14: (d)

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}} = \lim_{n\to\infty} \frac{n}{n\sqrt{1+\frac{1}{n}}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}} = 1$$

Sol–15: (a)

Let $y = \lim_{x\to a} (x-a)^{(x-a)}$

Taking logarithm on both sides.

$$\log y = \lim_{x\to a} \log (x-a)^{(x-a)}$$

$$= \lim_{x\to a} (x-a)\log(x-a)$$

$$= \lim_{x\to a} \frac{\log(x-a)}{\left(\frac{1}{x-a}\right)}$$

$$\text{Apply L'H Rule} = \lim_{x\to a} \frac{\frac{1}{(x-a)}}{-\frac{1}{(x-a)^2}}$$

$$\log y = \lim_{x\to a} -(x-a)$$

$$\log y = 0$$

$$\Rightarrow y = e^\circ = 1$$

Sol–16: (c)

$$\underset{x\to1}{\text{Lt}}\left(\frac{x^2-1}{x-1}\right) = \underset{x\to1}{\text{Lt}} \frac{(x-1)(x+1)}{(x-1)}$$

$$= \underset{x\to1}{\text{Lt}}(x+1) = 1 + 1$$

$$\boxed{\underset{x\to1}{\text{Lt}}\left(\frac{x^2-1}{x-1}\right) = 2}$$

Sol–17: (a)

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\left[1-\frac{a^4}{x^4}\right]$$

$$= 1-\frac{a^4}{(\infty)} = 1-0 = 1$$

Sol–18: (d)

$$f(x) = |x| = \begin{cases} x, & x>0 \\ -x, & x<0 \end{cases}$$

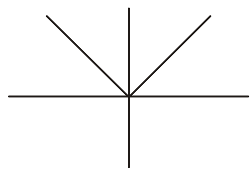
Left derivative = – 1

Right derivative = + 1

$\because$ LD $\neq$ RD

The derivative does not exist at x = 0.

Alternative Solution :



It is clear from graph f(x) is not differentiable at x = 0.

Sol–19: (d)

$$\lim_{x\to\frac{\pi}{4}} \cdot \frac{\sin\left[2\left(x-\frac{\pi}{4}\right)\right]}{\left(x-\frac{\pi}{4}\right)} \left(\frac{0}{0}\text{form}\right)$$

$$= \lim_{x\to\frac{\pi}{4}} \cdot \frac{2\cos\left[2\left(x-\frac{\pi}{4}\right)\right]}{1} = 2$$

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