ENGINEERING MATHEMATICS

(For ESE & GATE Exam)

(CE, ME, PI, CH, EC, EE, IN, CS, IT)

Salient Features :

- **280 topics under 32 chapters in 8 units**
- **640 Solved Examples for comprehensive understanding**
- **1772 Questions (last 33 years of GATE & 8 years of ESE Exams) with detailed solutions**
- **Only book having complete theory on ESE & GATE Pattern**
- **Comprising conceptual questions marked with '*' for quick revision**

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PREFACE

Science and technology has contributed greatly to the progress of the human race, and thus has made our life quite easier and comfortable. Engineers have played a vital role in the advancements of the technological world. However, with advancements in technology, there come challenges. It is here that the role of Engineering Mathematics comes in play to address the technological challenges posed by the modern era.

Engineering Mathematics is the art of applying mathematics to complicated real-world problems. It is a mix of mathematical theory, practical engineering and scientific computing.

Keeping in mind the importance of Engineering Mathematics skill in modern engineers, in the year 2017, the Union Public Service Commission (UPSC) introduced Engineering Mathematics as a common paper in the syllabus for Engineering Services Examination (ESE), and technical paper for Electrical (EE) stream. It has already been given a weightage of 15% in the Graduate Aptitude Test in Engineering (GATE)

With an objective to develop an ESE or GATE aspirant's numerical abilities and calculation skills, IES Master has come up with this Engineering Mathematics book that brings them face to face with **280 topics under 32 chapters in 8 units**, along with previous years questions from **GATE (last 33 years), and ESE (last 8 years)** and their detailed solutions. Equipped with all this, students can easily decide, how much time to allocate on each chapter based on the number of questions asked in that particular exam.

It is the only book in the market that includes complete theory exactly on the basis of ESE & GATE exams pattern. After each topic, the book includes more than **640 solved examples** for concept building & easy learning. To save students time during revision, all the previously asked conceptual questions are marked with '*' symbol.

Our special thanks to the entire IES Master team for their continuous support in bringing out the book. We strongly believe that this book will help students in their journey to success. Suggestions from students, teachers & educators for further improvement in the book are always welcome.

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CONTENTS

UNIT 1 : LINEAR ALGEBRA

UNIT 3 : VECTOR CALCULUS

UNIT 5 : DIFFERENTIAL EQUATIONS

UNIT 6 : NUMERICAL METHODS

UNIT 7 : TRANSFORM THEORY

UNIT 8 : PROBABILITY & STATISTICS

Functions and their Graphs

2.1

FUNCTIONS

A relation R from set A to B is said to be a function (f) if every element of set A has one and only one image in set B. It is denoted by $f : A \to B$, where A = domain of function and B = the co-domain of function.

The elements of set B which are the image of the elements of set A are called range.

Let $f : N \to N$ such that f(n) = 2x

then domain = N,

range = even natural no. = {2, 4, 6, 8, ……},

co-domain = N

TYPES OF FUNCTIONS

1. **One-One function (Injective) :** A one-to-one function is a function in which the answer never repeat. A normal function can have two different input values that produce the same answer, but a one-two-one function does not.
   i.e. if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ then f is 'one-one'.
   Otherwise the function is 'many one'.
2. **Onto function (Surjective) :** A function $f : X \to Y$ is said to be onto (surjective) if every element of Y is the image of some element of X under f. i.e., for every element $y \in Y$, there exists an element $x \in X$ such that f(x) = y. Otherwise the function is INTO.
3. **Bijective function :** A function which is both one-one and onto is called bijective function/one one correspondence.

Example 1 : Show that the function $f : R \to R$ defined as f(x) = 3x + 4 is bijective.

Solution : **For One-One :** Consider $x_1, x_2 \in R$ (Domain)

$$\text{If } f(x_1) = f(x_2)$$

$$3x_1 + 4 = 3x_2 + 4$$

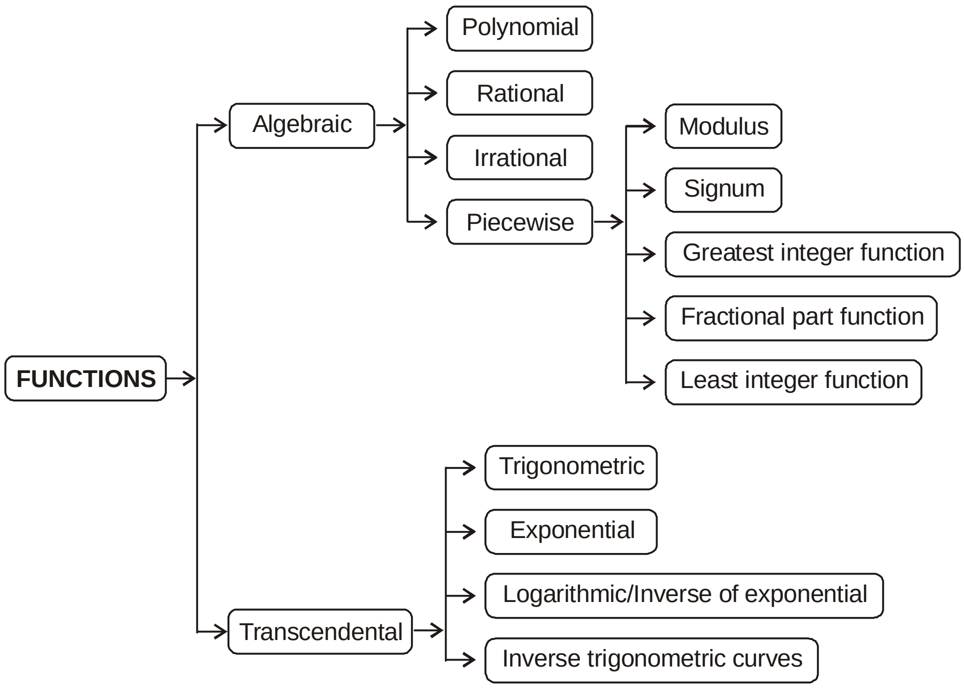
$$3x_1 = 3x_2 \Rightarrow x_1 = x_2 \text{ so f is One-One.}$$

For Onto : Consider $y \in R$ (Co-domain)

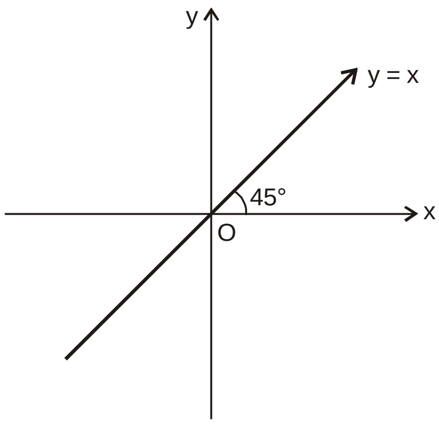
then $f(x) = 3x + 4 \Rightarrow y = 3x + 4$ or $x = \frac{y-4}{3}$ $\quad \because y \in R$ then $x \in R$

So, for every element $y \in R$ (co-domain) there exist an element $x \in R$ such that f(x) = y. So, f is Onto.

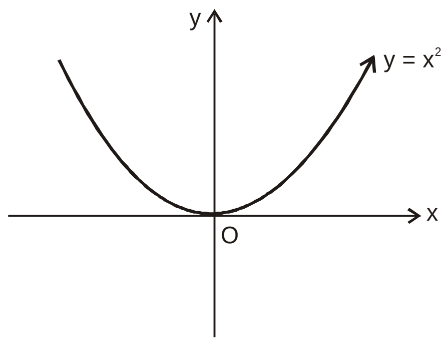
SOME BASIC GRAPHS



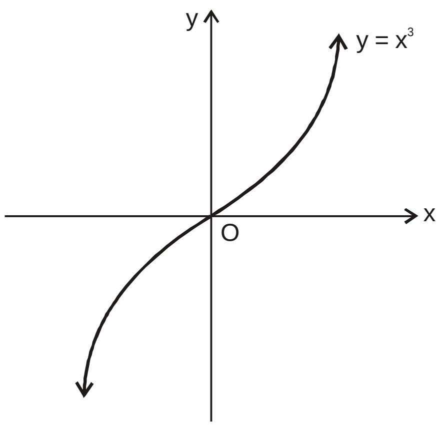
Graph of f(x) = x



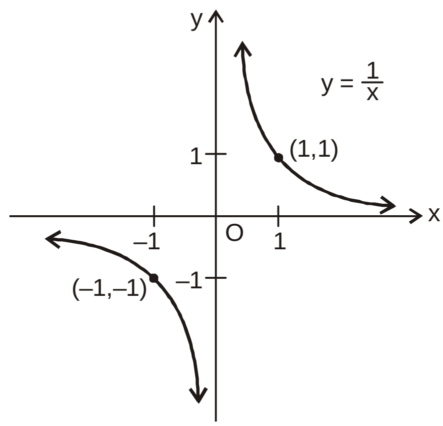
Graph of $f(x) = x^2$



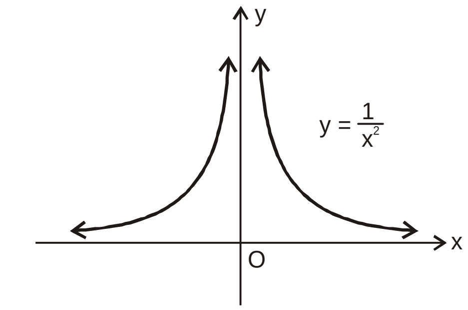
Graph of $f(x) = x^3$



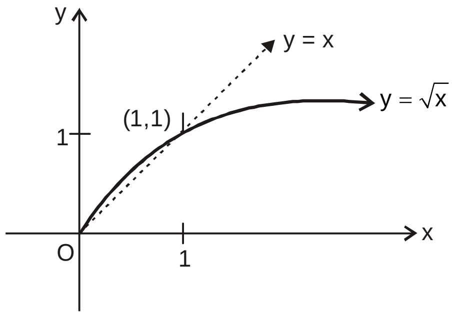
Graph of $f(x) = \frac{1}{x}$



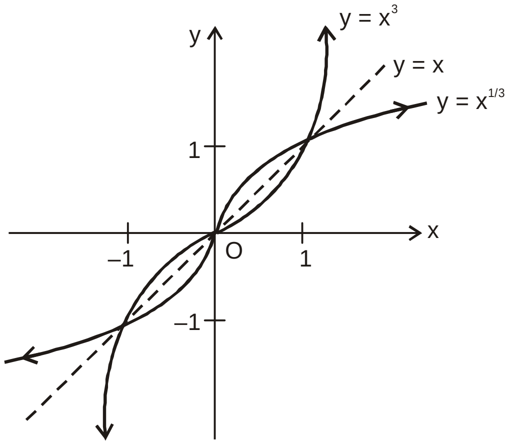
Graph of $f(x) = \frac{1}{x^2}$



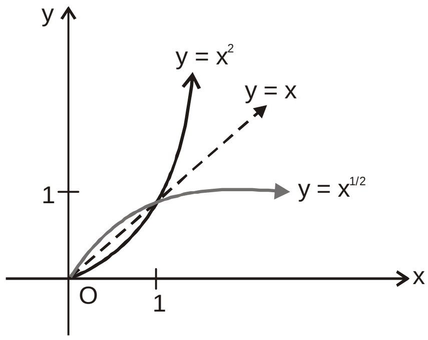
Graph of $f(x) = x^{1/2}$



Graph of $f(x) = x^{1/3}$

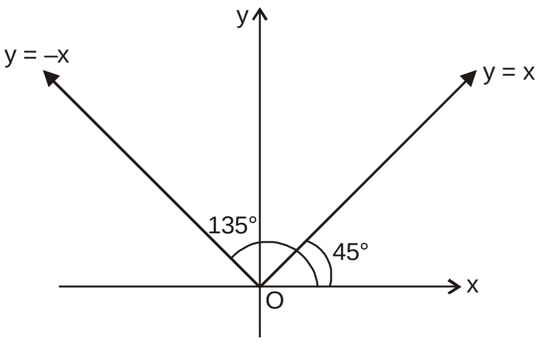


Graph of $f(x) = x^{1/2}$



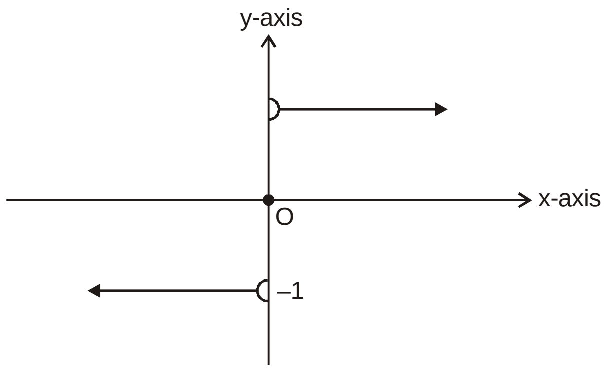
Modulus Function

Graph of $f(x) = |x| = \begin{cases} -x & \vdots\ x < 0 \\ 0 & \vdots\ x = 0 \\ x & \vdots\ x > 0 \end{cases}$



Signum Function

Graph of $f(x) = \text{sgn}(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} -1 & \vdots\ x < 0 \\ 1 & \vdots\ x > 0 \\ 0 & \vdots\ x = 0 \end{cases}$



(Greatest Integer Function)

Graph of $f(x) = [x] = \begin{cases} -2, & -2 \le x < -1 \\ -1, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$

(Fractional Part Function)

Graph of $f(x) = \{x\} = x - [x] = \begin{cases} x+2 & -2 \le x < -1 \\ x+1 & -1 \le x < 0 \\ x & 0 \le x < 1 \\ x-1 & 1 \le x < 2 \end{cases}$

Infinite Series

2.2

EXPANSION OF FUNCTIONS

We can expand some functions of x in the ascending powers of x as infinite series. For this, Maclaurin's series and Taylor's series are used in general.

MACLAURIN'S THEOREM (SERIES)

Let f(x) be a function of x which can be expanded in powers of x and let it is differentiable term by term any number of times, then f(x) can be expanded in the neighboured of x = 0 as follows

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots + \frac{x^n}{n!}f^{(n)}(0) + \ldots$$

Important Maclaurin's Expansion

1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\infty$
2. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\infty$
3. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\infty$
4. $\tan x = x + \frac{x^2}{3} + \frac{2}{15}x^3 + \ldots\infty$
5. $\sin h\, x = \frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\infty$
6. $\cosh x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\infty$
7. $\tanh x = x - \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots\infty$

TAYLOR'S THEOREM (SERIES)

Let f(x + h) be a function of x, which can be expanded in powers of h, and let this expansion is differentiable any number of times with respect to x, then f(x) can be expanded in the neighbourhood of x = a as follows:

$$f(a + h) = \frac{f(a)}{0!} + \frac{hf'(a)}{1!} + \frac{h^2}{2!}f''(a) + \ldots + \frac{h^n}{n!}f^{(n)}(a) + \ldots \text{ (where } x = a + h)$$

$$f(x) = \frac{f(a)}{0!} + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \ldots.$$

Failure of Taylor's Theorem

Taylor's theorem fails if any one of the following cases arises:

(i) f(x) or any one of its derivativs, becomes infinite between the values of the given variable.

(ii) f(x) or any one of its derivatives becomes discontinuous between the same values.

(iii) The remainder $\frac{h^n}{n!}f^n(a + \theta_n h)$ tends to non-zero value as n tends to infinite and in this case the series $\sum \frac{h^n}{n!}f^n(x)$ is divergent.

Example 1 : Expand ln(1 + x) by Maclaurin's theorem.

Solution : Let f(x) = ln(1 + x) $\Rightarrow$ f(0) = 0 …(i)

Differentiating successively, we get

$$\Rightarrow \quad f'(x) = \frac{1}{1+x} \Rightarrow f'(0) = 0$$

$$f''(x) = \frac{d}{dx}(1+x)^{-1} = -(1+x)^{-2} = -\frac{1}{(1+x)^2} \Rightarrow f''(0) = -1$$

$$f'''(x) = \frac{d}{dx}[-(1+x)^{-2}] = \frac{2}{(1+x)^3} \Rightarrow f'''(0) = 2$$

$$f^{(iv)}(x) = \frac{d}{dx}[2(1+x)^{-3}] = -\frac{6}{(1+x)^4} \Rightarrow f^{(iv)}(0) = -6, \ldots$$

By Maclaurin's Theorem, we have f(x) = $f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$ …(ii)

Put all the values in equation (ii), we get f(x) = $0 + x\cdot 1 + \frac{x^2}{2!}(-1) + \frac{x^3}{3!}(2) + \frac{x^4}{4!}(-6) + \ldots$

$$\Rightarrow \quad f(x) = \ln(1+x) = x - \frac{x^2}{2!} + \frac{2x^3}{3!} - \frac{6x^4}{4!} + \ldots = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

Example 2 : Expand log cosx using Maclaurin's theorem upto the coefficient of x^4.

Solution : Let f(x) = $\ln\cos x \Rightarrow f(0) = 0$ …(i)

Differentiating above function successively, we get

$$f'(x) = \frac{1}{\cos x}(-\sin x) = -\tan x \Rightarrow f'(0) = 0$$

$$f''(x) = \frac{d}{dx}(-\tan x) = -\sec^2 x \Rightarrow f''(0) = -1$$

$$f'''(x) = \frac{d}{dx}(-\sec^2 x) = -2\sec^2 x\tan x \Rightarrow f'''(0) = 0$$

$$f^{(iv)}(x) = \frac{d}{dx}\left(-2\sec^2 x\tan x\right) = (-4\sec^2 x\tan^2 x - 2\sec^4 x) \Rightarrow f^{(iv)}(0) = -2$$

By Maclaurin's theorem, we have

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{(iv)}(0) + \ldots$$

Put all the values, we get

$$f(x) = \ln\cos x = 0 + x(0) + \frac{x^2}{2!}(-1) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-2) + \ldots = -\left(\frac{x^2}{2!} + \frac{2x^4}{4!} + \ldots\right)$$

Example 3 : Expand $\sin\left(\frac{\pi}{4} + \theta\right)$ in powers of θ.

Solution : Let

$$f(x) = \sin x \Rightarrow f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad f'(x) = \cos x \Rightarrow f'\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad f''(x) = -\sin x \Rightarrow f''\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad f'''(x) = -\cos x \Rightarrow f'''\left(\frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}} \ldots$$